

Pulse tube stirling machine with warm gas-driven displacer

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ABSTRACT

A pulse tube type stirling machine with warm gas-driven displacer which has a displacer rod is discussed with numerical simulation when it is used as a cryogenic refrigerator, room temperature refrigerator and engine. It has both the advantages of gas-driven stirling machine with high efficiency and simplicity and the advantages of pulse tube machine with no moving parts at low temperatures. A nodal analysis method that includes the linear motor and the displacer in the machine is introduced. Numerical results show that it has high potential to be used as the cryogenic refrigerator, room temperature refrigerator and engine. In this type of machine, there is an optimum phase angle between displacer and piston, and an optimum swept volume ratio of displacer over compressor for efficiency. The phase angle and swept volume ratio can be adjusted by the natural frequency of the displacer and the diameter of the displacer rod when it is used as a refrigerator.

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1. Introduction

The advantage of the common pulse tube refrigerators, such as orifice, double inlet, and inertance tube pulse tube refrigerators, is that there is no moving part at low temperatures and only one moving part at room temperature [1–5]. The expansion work in the pulse tube of the refrigerator is changed to heat, therefore the theoretical efficiency is lower compared to a Carnot refrigerator. In a warm piston or warm displacer type pulse tube refrigerator [6–11] besides a compressor as moving part at room temperature, there is a piston or a displacer at the warm end of the pulse tube to recover the expansion work. Though it is complex, the theoretical efficiency is the same as that of a Carnot refrigerator if all else is perfect, and any phase angle between the pressure and the flow can be achieved to minimize the regenerator loss [8]. In this paper, a nodal analysis numerical method was introduced for a pulse tube machine with a warm gas-driven displacer which has a displacer rod [11], and the machine was simulated by a numerical method when it is used as cryogenic refrigerator, room temperature refrigerator, and engine. The numerical method is similar to that in Ref. [5], and the motions of the linear motor and the displacer are included.

2. Structure of the novel machine

Fig. 1 shows the schematic of the pulse tube type stirling machine that has a displacer with a displacer rod as a phase shifter.

It includes a compressor, a displacer part and a fixed part. The fixed part includes a cooler, a regenerator, an absorption heat exchanger, a pulse tube, a connecting tube, and gas supply tube. The compressor includes a motor house, a linear motor, a group of flexible bearings and a piston. The displacer part includes a buffer, a group of flexible bearings, a displacer and a displacer rod.

The displacer with the rod is suspended by flexible bearing to form a clearance seal. The displacer with the rod and the flexible bearings form an oscillation system. The displacer oscillates due to the pressure oscillation in working spaces and the buffer. The expansion power of the pulse tube is obtained on the displacer expansion space and is transferred to the displacer back space that joins the compression space to compress the working gas.

When it is used as a refrigerator, cooling power is obtained from the absorption heat exchanger which is at a low temperature. The electric power is input to the linear motor. Compared to the inertance tube pulse tube refrigerator [5], this machine has a higher efficiency because the expansion power from the pulse tube is recovered by the displacer. When operating as a room temperature refrigerator, the advantage is even larger because the expansion power from the pulse tube is not so small compared to the input power of the compressor.

When it is used as an engine, the heat is supplied by the absorption heat exchanger which is kept at a high temperature. The linear motor becomes a linear generator from which electric power can be obtained. Compared to a stirling engine, the advantage of this type of machine is that there is no moving part at high temperature, and the moving parts are at room temperature and can be separated from the fixed parts.

In Fig. 1, the displacer is driven by gas pressure difference. Only the linear motor needs a sinusoidal wave current which can be

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Nomenclature

a	swept volume ratio	R	gas constant
A	cross gas flow area	R_m	motor coil resistance
A_s	cross solid area	s	enthalpy
A_L	heat transfer area per meter	S	enthalpy flow
B	motor force factor	t	time
C_V	constant volume specific heat of gas	T	temperature of gas
C_P	constant pressure specific heat of gas	T_0	room temperature
C_S	specific heat of solid	T_S	temperature of matrix
D_e	hydro-diameter	u	speed
D_p	diameter of piston	U	voltage
D_d	diameter of displacer	U_0	voltage amplitude
D_r	diameter of displacer rod	V_C	volume of compressor
E	exergy flow	V_{CS}	dead volume of compressor
f_r	friction coefficient	V_{C_0}	swept volume of compressor
f_p	resistance of piston	V_E	displacer volume
f_d	resistance of displacer	V_{ES}	displacer dead volume
F_p	piston clearance leakage factor	V_{E_0}	displacer swept volume
F_d	rod clearance seal leakage factor	V_{EB}	displacer back volume
H	enthalpy flow	W	compressor power
H_{PT}	pulse tube enthalpy flow	W_m	motor input power
H_{REG}	regenerator loss	x	coordinator along the length
i	current	x_p	displacement of piston
K_p	stiffness of piston spring	x_d	displacement of displacer
K_d	stiffness of displacer spring	x_{p_0}	distance between piston head and cylinder head at standstill
L	inductance of linear motor	x_{d_0}	distance between displacer head and displacer cylinder head at standstill
$\dot{m}$	mass flow rate		
$\dot{m}_p$	gas leakage from piston seal		
$\dot{m}_d$	gas leakage from rod seal		
m_s	mass of solid		
m_p	mass of piston and mover of linear motor		
m_d	mass of displacer		
P	pressure		
P_p	pressure in compressor cylinder		
P_d	pressure in displacer cylinder		
P_{Bp}	pressure in motor house		
P_{Bd}	pressure in buffer		
Q	cooling power or heating power		
		<i>Greek letters</i>	
		α	heat transfer coefficient
		η	engine efficiency
		ρ	density of gas
		ρ_S	density of matrix
		τ	period
		ϕ	phase angle between compressor and displacer
		ω	angle frequency

Greek letters

α	heat transfer coefficient
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provided by an AC power source or a simple inverter. There is no moving part at low temperature or high temperature range. In this sense the machine has the same advantage of both a pulse tube machine and a gas driven-stirling machine. We can call this ma-

chine a “pulse tube stirling machine with warm gas-driven displacer” or, alternatively, a “stirling machine with pulse tube”.

In Fig. 1, the buffer does not consume any work theoretically if the gas spring loss, friction loss, and gas leakage loss from the clearance seal around displacer rod are not considered.

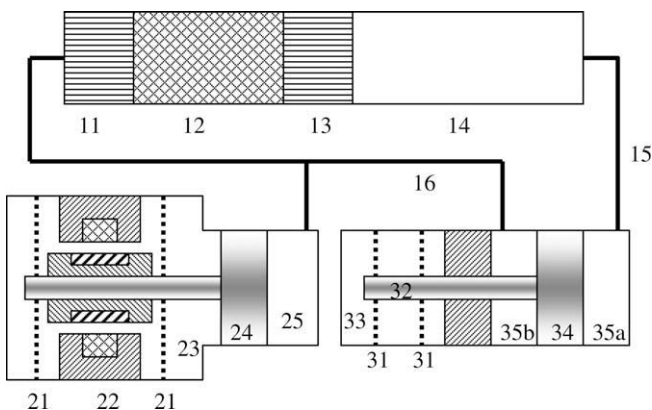


Fig. 1. Schematic drawing of the machine: 11. cooler; 12. regenerator; 13. adsorption heat exchanger; 14. pulse tube; 15. connecting tube; 16. gas supply tube; 21. compressor spring; 22. linear motor; 23. motor house; 24. piston; 25. compression space; 31. displacer spring; 32. rod; 33. buffer; 34. displacer; 35a. expansion space; 35b. back space.

3. Numerical simulation method

The numerical simulation is separated into two steps. In step one, the numerical model does not include the motion of the linear motor and the displacer. The motions of the displacer and the piston are assumed as sinusoidal to investigate the basic character. In step two, the numerical model includes the motion of the linear motor and the displacer to investigate how the displacer moves in the cryogenic refrigerator.

Assuming that the volume of the gas supply tube connecting the back space of the displacer and compression space of the compressor and the cooler is neglected, the compression space and the back space of the displacer are treated as single volume.

3.1. Sinusoidal displacement of piston and displacer

The numerical simulation was done by a nodal analysis method based on Ref. [5], and was mainly based on the assumptions that

the system is one dimensional, and that the working gas is an ideal gas. The governing equations are as follows.

Continuity equation

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial x}(\rho u A) = 0 \quad (1)$$

Momentum equation

$$\frac{\partial P}{\partial x} + \frac{f_r}{D_c} \frac{1}{2} \rho u^2 \frac{u}{|u|} = 0 \quad (2)$$

Energy equation for gas

$$\frac{\partial}{\partial t}(\rho A C_v T) + \frac{\partial}{\partial x}(\rho u A C_p T) + \alpha A_L(T - T_s) + \frac{P}{dx} \frac{dV}{dt} = 0 \quad (3)$$

Energy equation for matrix

$$A_S \rho_S C_S \frac{\partial T_s}{\partial t} = \alpha A_L(T - T_s) \quad (4)$$

Ideal gas state equation

$$P = \rho R T \quad (5)$$

Compression volume of the compressor

$$V_C = V_{CS} + 0.5 V_{C_0}(1 + \cos(\omega t - \varphi)) \quad (6)$$

Expansion volume of the displacer

$$V_E = V_{ES} + 0.5 V_{E_0}(1 + \cos(\omega t)) \quad (7)$$

Back volume of the displacer

$$V_{EB} = V_{ES} + 0.5 V_{E_0}(1 - \cos(\omega t)) \quad (8)$$

Swept volume ratio of the displacer over the compressor

$$a = \frac{V_{E_0}}{V_{C_0}} \quad (9)$$

Compression power of the compressor

$$W = \frac{1}{\tau} \oint P dV_C \quad (10)$$

Enthalpy flow

$$H = \frac{1}{\tau} \oint \dot{m} C_p T dt \quad (11)$$

Entropy flow

$$S = \frac{1}{\tau} \oint \dot{m} s dt \quad (12)$$

Exergy flow

$$E = H - S T_0 \quad (13)$$

Cooling power

$$Q = H_{PT} - H_{REG} \quad (14)$$

COP of refrigerator

$$\text{COP} = Q/W \quad (15)$$

Efficiency of engine

$$\eta = W/Q \quad (16)$$

The equivalent P - x diagram in the pulse tube definition is the same as that in Ref. [5].

3.2. Equations of motion for piston and displacer

As shown in Fig. 1, the linear motor drives the piston oscillation. The electric force of the linear motor is from the electric current. The forces on the piston are from the motor, the flexible spring,

the friction and the gas pressure difference on the two ends of the piston. The forces on the displacer are from the flexible spring, friction, the gas pressure difference between the buffer and the pulse tube on the rod, and the gas pressure difference on two ends of the displacer. Assuming that the friction factor of the clearance is constant for the gas leakage through the clearance seal, and that the electric force of the linear motor is proportional to the electric current, the following equations are obtained.

Differential equation of the motion of the piston

$$m_p \ddot{x}_p + f_p \dot{x}_p + k_p x_p = B i + \frac{\pi}{4} D_p^2 (P_{Bp} - P_p) \quad (17)$$

Differential equation of the motion of the displacer

$$m_d \ddot{x}_d + f_d \dot{x}_d + k_d x_d = \frac{\pi}{4} (D_d^2 - D_r^2) (P_p - P_d) + \frac{\pi}{4} D_r^2 (P_{Bd} - P_d) \quad (18)$$

Equation of electrodynamic driver

$$U = U_0 \sin(\omega t) = B \dot{x}_p + i R_m + L \frac{di}{dt} \quad (19)$$

The gas leakage through the clearance seal between the compression volume and the motor house

$$\dot{m}_p = \begin{cases} F_p \sqrt{(P_{Bp}^2 - P_p^2)} \\ -F_p \sqrt{(P_p^2 - P_{Bp}^2)} \end{cases} \quad (20)$$

The gas leakage through the clearance seal between the back volume and the buffer

$$\dot{m}_d = \begin{cases} F_d \sqrt{(P_{Bd}^2 - P_p^2)} \\ -F_d \sqrt{(P_p^2 - P_{Bd}^2)} \end{cases} \quad (21)$$

Motor input power

$$W_m = \frac{1}{\tau} \oint U_i dt \quad (22)$$

With the above equations, the simulation of the displacement of the piston and the displacer can be achieved. Then the volume V_C , V_E , and V_{EB} in Eqs. (6)–(8) can be replaced by the following equations.

Compression volume of the compressor

$$V_C = \frac{\pi}{4} D_p^2 (x_{p0} - x_p) \quad (23)$$

Expansion volume of the displacer

$$V_E = \frac{\pi}{4} D_d^2 (x_{d0} - x_d) \quad (24)$$

Back volume of the displacer

$$V_{EB} = \frac{\pi}{4} (D_d^2 - D_r^2) (x_{d0} + x_d) \quad (25)$$

Table 1

Calculation condition of cryogenic refrigerator.

Regenerator	Ø120 mm × 50 mm, porosity 0.7, mesh wire diameter 0.04 mm
Pulse tube	Ø40 mm × 500 mm
Cooler height, width, length	0.33 mm × 2837.25 mm × 40 mm
Absorption heat exchanger height, width, length	0.5 mm × 829.38 mm × 40 mm
Compressor swept volume	500 cm ³
Refrigeration temperature	77 K
Room temperature	300 K
Frequency	50 Hz
Charge pressure of helium gas	2.5 MPa

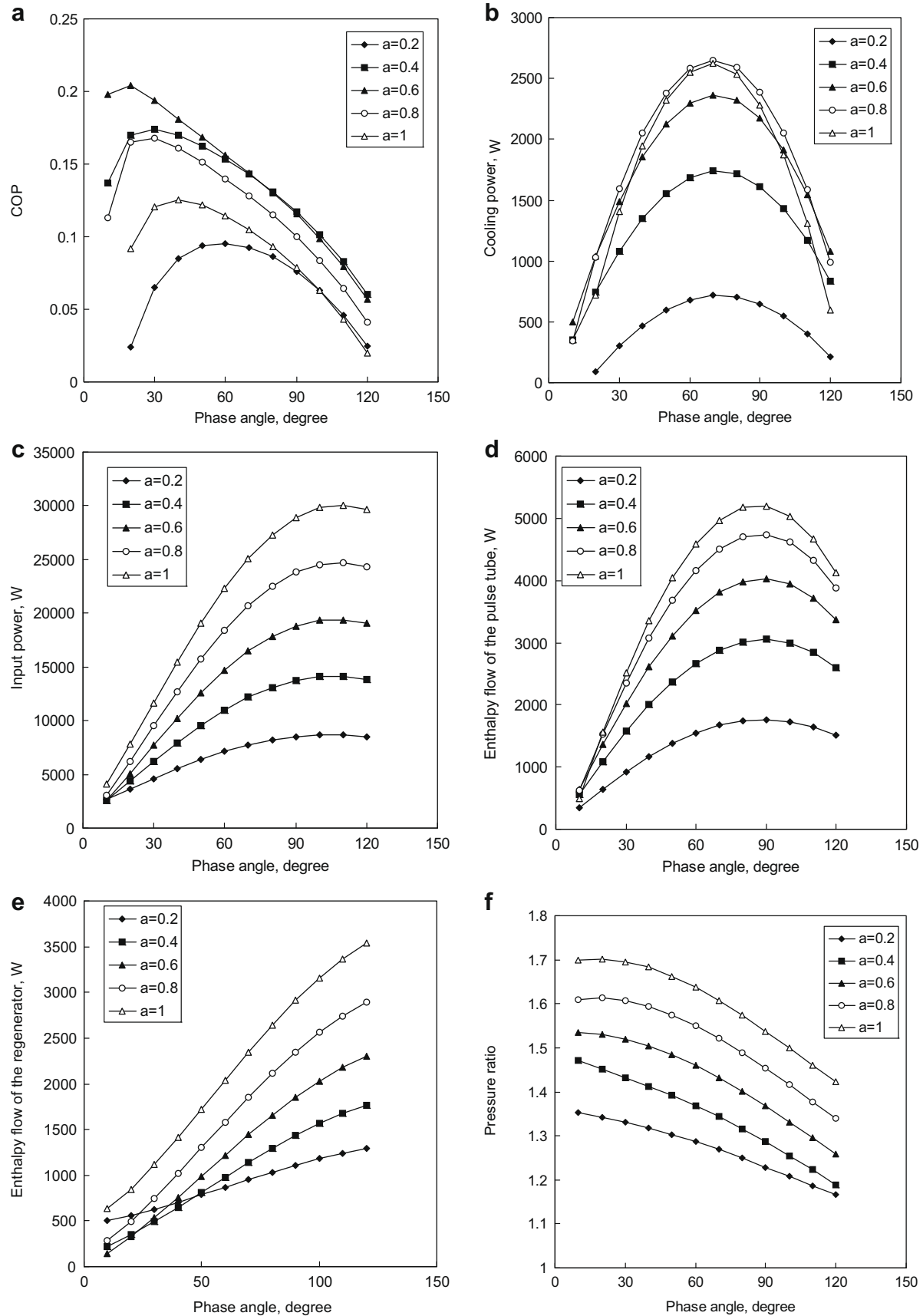


Fig. 2. (a) COP vs. phase angle and swept volume ratio. (b) Cooling power vs. phase angle and swept volume ratio. (c) Input power vs. phase angle and swept volume ratio. (d) Enthalpy flow of pulse tube vs. phase angle and swept volume ratio. (e) Enthalpy flow of regenerator vs. phase angle and swept volume ratio. (f) Pressure ratio in pulse tube vs. phase angle and swept volume ratio.

4. Numerical results with sinusoidal displacement

In this section, the numerical simulation with sinusoidal displacement of compressor and displacer is conducted when the pulse tube stirling machine is used as cryogenic refrigerator, as room temperature refrigerator, or as engine.

4.1. Cryogenic refrigerator

In this section, a kilowatt order refrigerator was simulated. With given fixed parts, the swept volume ratio of the displacer to the compressor, the phase angle between the compressor and the displacer are investigated. This is the basic information required for the design of this type of refrigerator.

Table 1 shows the basic data of the refrigerator for the simulation. With this condition, Fig. 2 shows the COP, cooling power, input power, pressure ratio, enthalpy flow of the pulse tube and enthalpy flow of the regenerator change with the swept volume ratio of the displacer to the compressor and the phase angle between the compressor and the displacer. In Fig. 2, the swept volume ratios are 0.2, 0.4, 0.6, 0.8 and 1. There is an optimum phase angle for each of COP, cooling power, input power, and the enthalpy flow

of the pulse tube. The optimum phase angle for cooling power is larger than that for the COP, and is smaller than that for the input power. The enthalpy flow of the regenerator increases with the increase of the phase angle. There is an optimum swept volume ratio of the displacer to the compressor for COP and cooling power. In the calculation range, the input power and the enthalpy flow of the pulse tube increase with the increase of the volume ratio of the displacer to the compressor. When the phase angle is smaller than 50° , there is an optimum swept volume ratio for minimum enthalpy flow of the refrigerator. When the phase angle is over 50° , the enthalpy flow of the regenerator increases with the increase of the swept volume ratio of the displacer to the compressor. The pressure ratio decreases with the increase of the phase angle and increases with the increase of the volume ratio of the displacer to the compressor.

It should be mentioned that the COP is over 0.2, which indicates this machine could get higher efficiency than that of the inertance tube pulse tube refrigerator [5]. One of the reasons is that the expansion power of the pulse tube is recovered by the displacer. Another reason may be the high pressure ratio. At optimum COP point, the pressure ratio is 1.5. High pressure ratio also means this type of refrigerator has high power density.

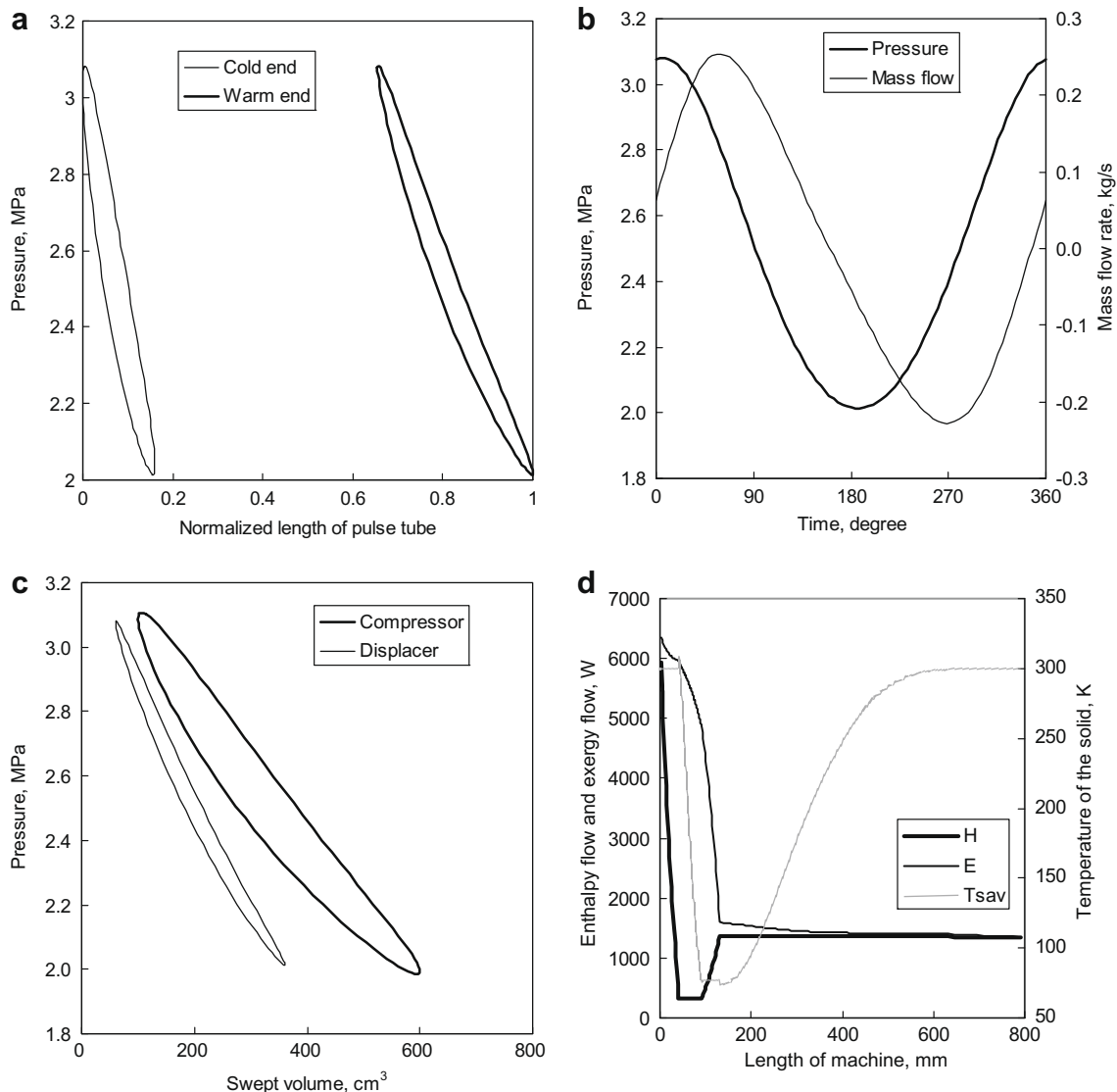


Fig. 3. (a) P-x diagrams in pulse tube. (b) Mass flow rate and pressure wave at the cold end of regenerator. (c) PV diagrams of displacer and compressor. (d) Enthalpy flow, exergy flow and temperature distribution in the machine.

Fig. 3 shows the P - x diagrams in the pulse tube, the pressure wave and the mass flow rate at the cold end of the pulse tube, the PV diagrams of compressor and displacer, enthalpy flow, exergy flow, and temperature distribution of the machine at optimum COP point when the phase angle is 20° and swept volume ratio is 0.6. There is a long distance between the P - x diagrams, which indicates that the pulse tube length is large enough. Sufficient distance between the P - x diagrams at the cold end and warm end of the pulse tube means there is a sufficient separation between cold gas and warm gas. If this were not the case, cold gas and warm gas flowing to the same place may generate large shuttle loss in the pulse tube. The pressure is ahead of a large phase angle to mass flow rate, which indicates high efficiency condition of the regenerator. The PV diagram of the displacer is not so small compared to the PV diagram of the compressor, which indicates that the gain from the displacer is not so small. In Fig. 3d, the flat line of enthalpy

flow indicates the regenerator and the pulse tube. The flat line of temperature indicates the cooler, absorption heat exchanger, and connecting tube. The exergy flow decreases from left to right and remains almost constant in the pulse tube to the hot end where it is taken off by the displacer.

Fig. 4 shows the COP and cooling power changes with swept volume of the compressor under the condition of 20° of the phase angle between the compressor and the displacer and 0.6 of swept volume ratio of the displacer over the compressor. In the calculation range, with the increase of the compressor swept volume, the cooling power, input power, pressure ratio increase, and there is an optimum point for COP.

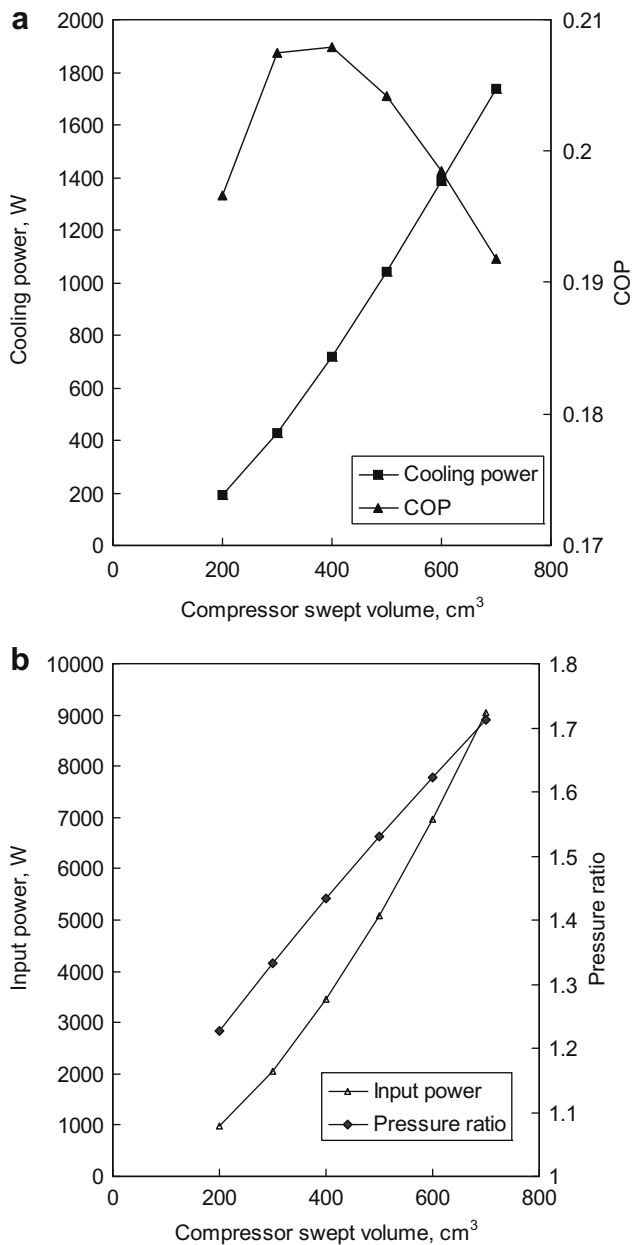


Fig. 4. (a) COP and cooling power vs. swept volume of compressor. (b) Input power and pressure ratio vs. swept volume of compressor.

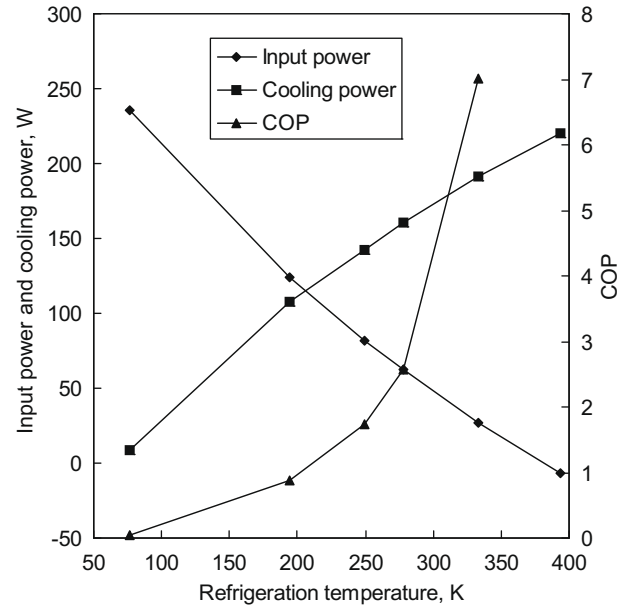


Fig. 5. Cooling power, input power and COP of room temperature refrigerator.

Table 2
Calculation condition of room temperature refrigerator.

Regenerator	$\varnothing 30 \text{ mm} \times 30 \text{ mm}$, porosity 0.7449, mesh wire diameter 0.05 mm
Pulse tube	$\varnothing 20 \text{ mm} \times 120 \text{ mm}$
Cooler height, width, length	0.33 mm $\times$ 353.43 mm $\times$ 20 mm
Absorption heat exchanger height, width, length	0.33 mm $\times$ 353.43 mm $\times$ 20 mm
Compressor swept volume	15 cm ³
Displacer swept volume	15 cm ³
Phase angle	30°
Room temperature	333.15 K
Frequency	50 Hz
Charge pressure of helium gas	3 MPa

Table 3
Calculation condition of engine.

Regenerator	$\varnothing 30 \text{ mm} \times 60 \text{ mm}$, porosity 0.7, mesh wire diameter 0.1 mm
Pulse tube	$\varnothing 30 \text{ mm} \times 200 \text{ mm}$
Cooler height, width, length	0.33 mm $\times$ 353.43 mm $\times$ 40 mm
Absorption heat exchanger height, width, length	0.33 mm $\times$ 353.43 mm $\times$ 40 mm
Compressor swept volume	50 cm ³
Heater temperature	900 K
Room temperature	300 K
Frequency	100 Hz
Charge pressure of helium gas	3 MPa

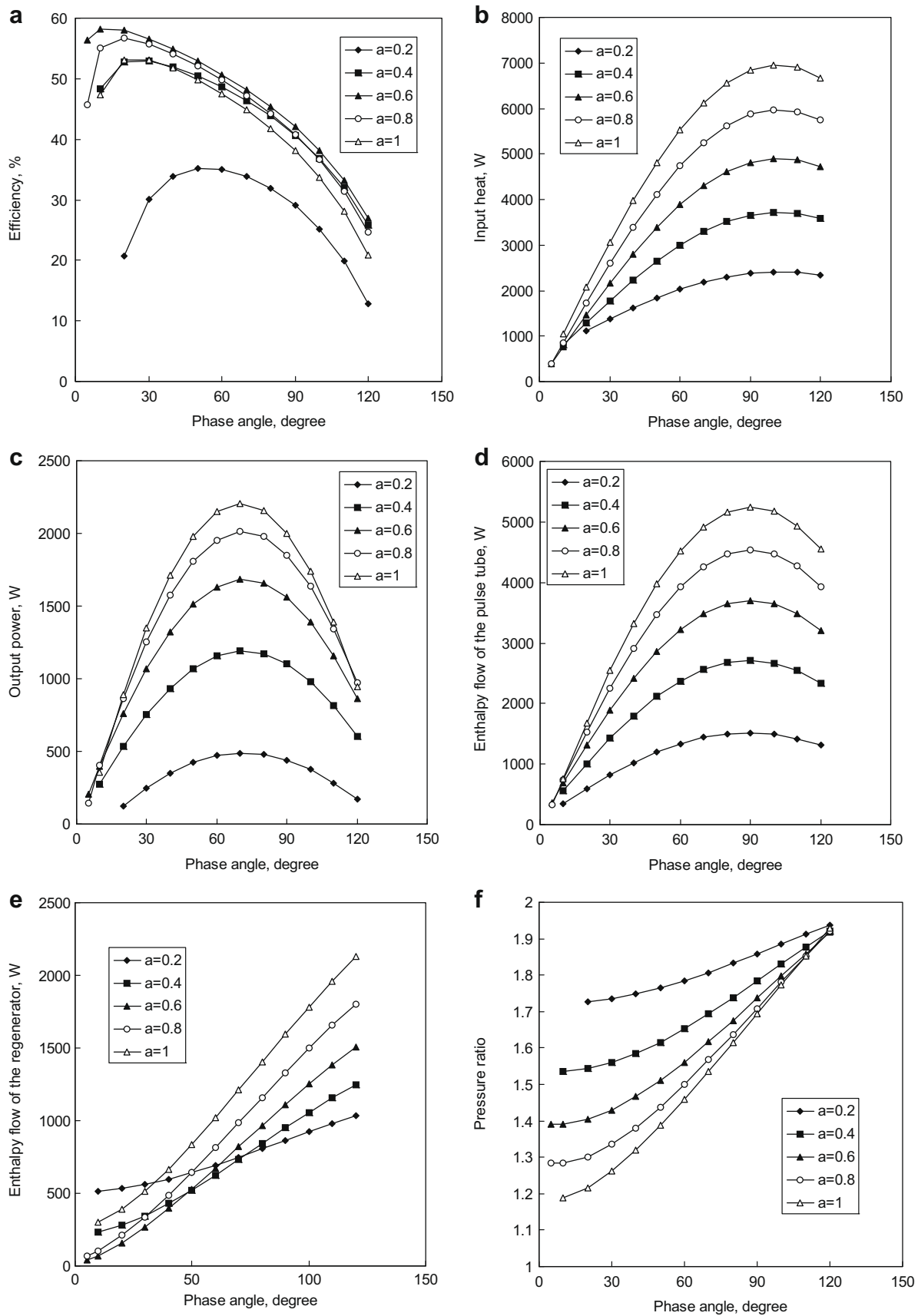


Fig. 6. (a) Efficiency vs. swept volume ratio and phase angle. (b) Input heat vs. swept volume ratio and phase angle. (c) Output power vs. swept volume ratio and phase angle. (d) Enthalpy flow of pulse tube vs. swept volume ratio and phase angle. (e) Enthalpy flow of regenerator vs. swept volume ratio and phase angle. (f) Pressure ratio in pulse tube vs. swept volume ratio and phase angle.

The above results show that there is an optimum swept volume ratio of the displacer to the compressor and an optimum phase angle between the compressor and the displacer for COP at a given swept volume of the compressor.

4.2. Room temperature refrigerator

Fig. 5 shows the input power, cooling power and COP change with the refrigeration temperature under the conditions shown in Table 2. The COP and cooling power increase with the increase in refrigeration temperature, and the input power decreases with the increase in refrigeration temperature. Near room temperature the COP is larger than 2. A COP of 2 can not be achieved by the iner-tance tube pulse tube refrigerator because its theoretical efficiency is less than unity.

In Fig. 5, the COP is 7 when the temperature of the absorption heat exchanger is the same as the room temperature. Theoretically, the machine works as an engine when the temperature of the absorption heat exchanger is higher than that of the cooler. The machine generates an output power of 7 W when the temperature of the absorption heat exchanger is 393.2 K. From the room tem-

perature to a point where the temperature of the absorption heat exchanger is higher than the room temperature and the output power is zero, the machine just consumes the input power, or the machine cannot move due to loss.

When the temperature of the absorption heat exchanger is the same as the room temperature and the temperature of the cooler is higher than the room temperature, the machine works as a heat pump.

This type of machine can be considered as the merge of pulse tube and stirling refrigerator. So it can be very small and be operated at a wide range of refrigeration temperatures.

4.3. Engine

In this section, a kilowatt order engine was simulated. With given fixed parts, the swept volume ratio of the displacer to the compressor, the phase angle between the compressor and the displacer are investigated. The calculation condition is shown in Table 3. Here, the higher frequency is used for higher power density.

Fig. 6 shows the input heat, efficiency, output power, pressure ratio, enthalpy flow of the pulse tube, and enthalpy flow of the

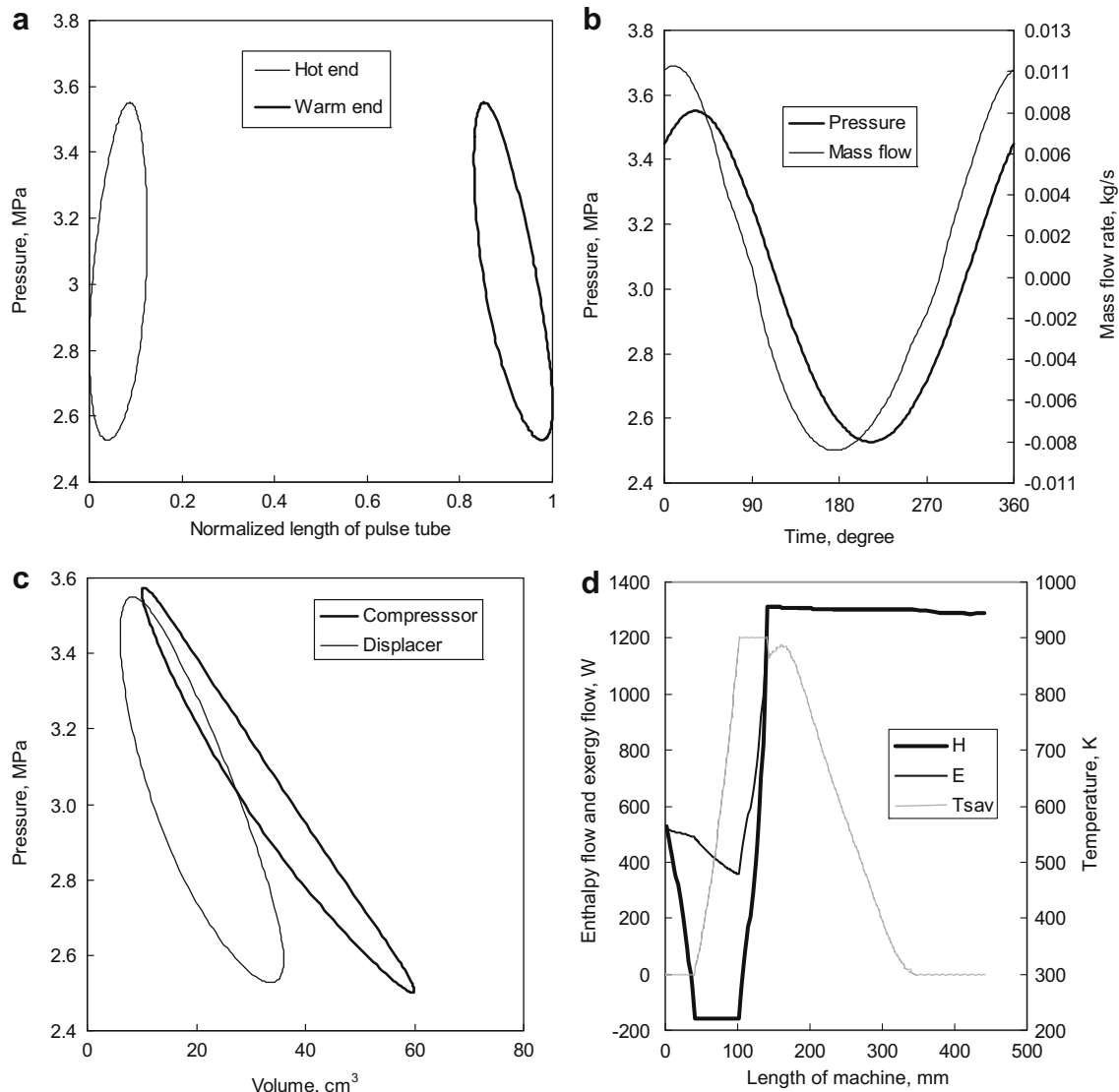


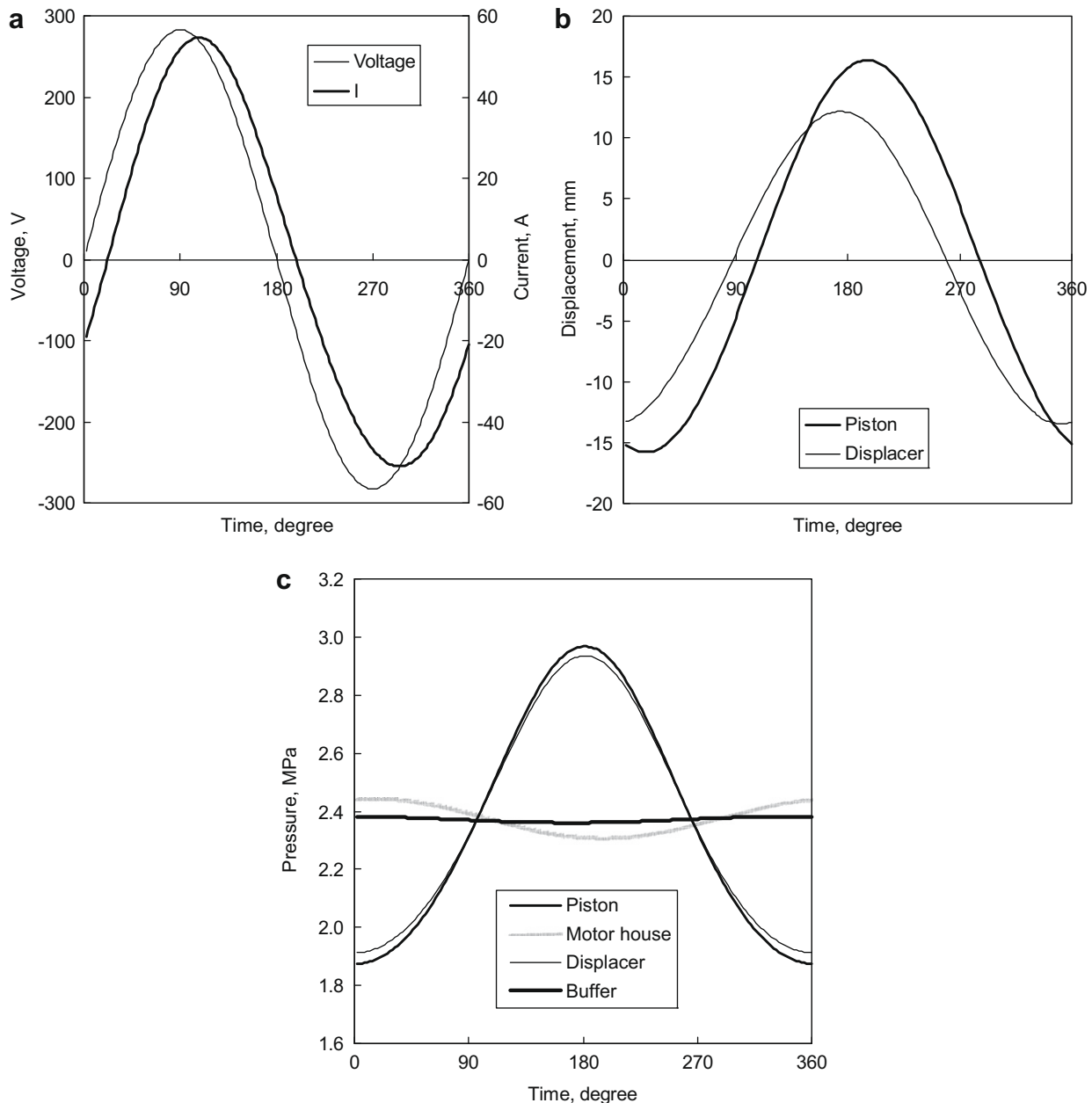
Fig. 7. (a) P-x diagrams in pulse tube. (b) Mass flow rate and pressure wave at the hot end of the pulse tube. (c) PV diagrams of displacer and compressor. (d) Enthalpy flow, exergy flow and temperature distribution in the machine.

Table 4

Calculation condition of cryogenic refrigerator.

Regenerator	∅120 mm × 50 mm, porosity 0.7, mesh wire diameter 0.04 mm
Pulse tube	∅40 mm × 500 mm
Cooler height, width, length	0.33 mm × 2837.25 mm × 40 mm
Absorption heat exchanger height, width, length	0.5 mm × 829.38 mm × 40 mm
Refrigeration temperature	77 K
Room temperature	300 K
Frequency	50 Hz
Charge pressure of helium gas	2.5 MPa
Voltage	283 V
Piston diameter	150 mm
Piston spring stiffness	500×10^3 N/m
Piston weight	11.5 kg
Displacer diameter	120 mm
Displacer spring stiffness	50×10^3 N/m
Displacer weight	0.5 kg
Displacer rod diameter	30 mm

regenerator change with the swept volume ratio of the displacer to the compressor and the phase angle between the compressor and the displacer. In the figure, the swept volume ratios are 0.2, 0.4, 0.6, 0.8 and 1. There is an optimum phase angle for efficiency, input heat, output power, and the enthalpy flow of the pulse tube. The optimum phase angle for output power is larger than that for efficiency, and is smaller than that of input heat. The enthalpy flow of the regenerator increases with the increase of the phase angle. There is an optimum swept volume ratio of the displacer over the compressor for efficiency. In the calculation range, the input heat, the output power and the enthalpy flow rate of the pulse tube increase with the increase of the volume ratio of the displacer to the compressor. When the phase angle is smaller than 70° , there is an optimum swept volume ratio for minimum enthalpy flow of the refrigerator. When the phase angle is over 70° , the enthalpy flow of the regenerator increases with the increase of the swept volume ratio of the displacer to the compressor. The pressure ratio

**Fig. 8.** (a) Voltage and current. (b) Displacement of displacer and piston. (c) Pressure in compressor, displacer, motor house and displacer buffer.

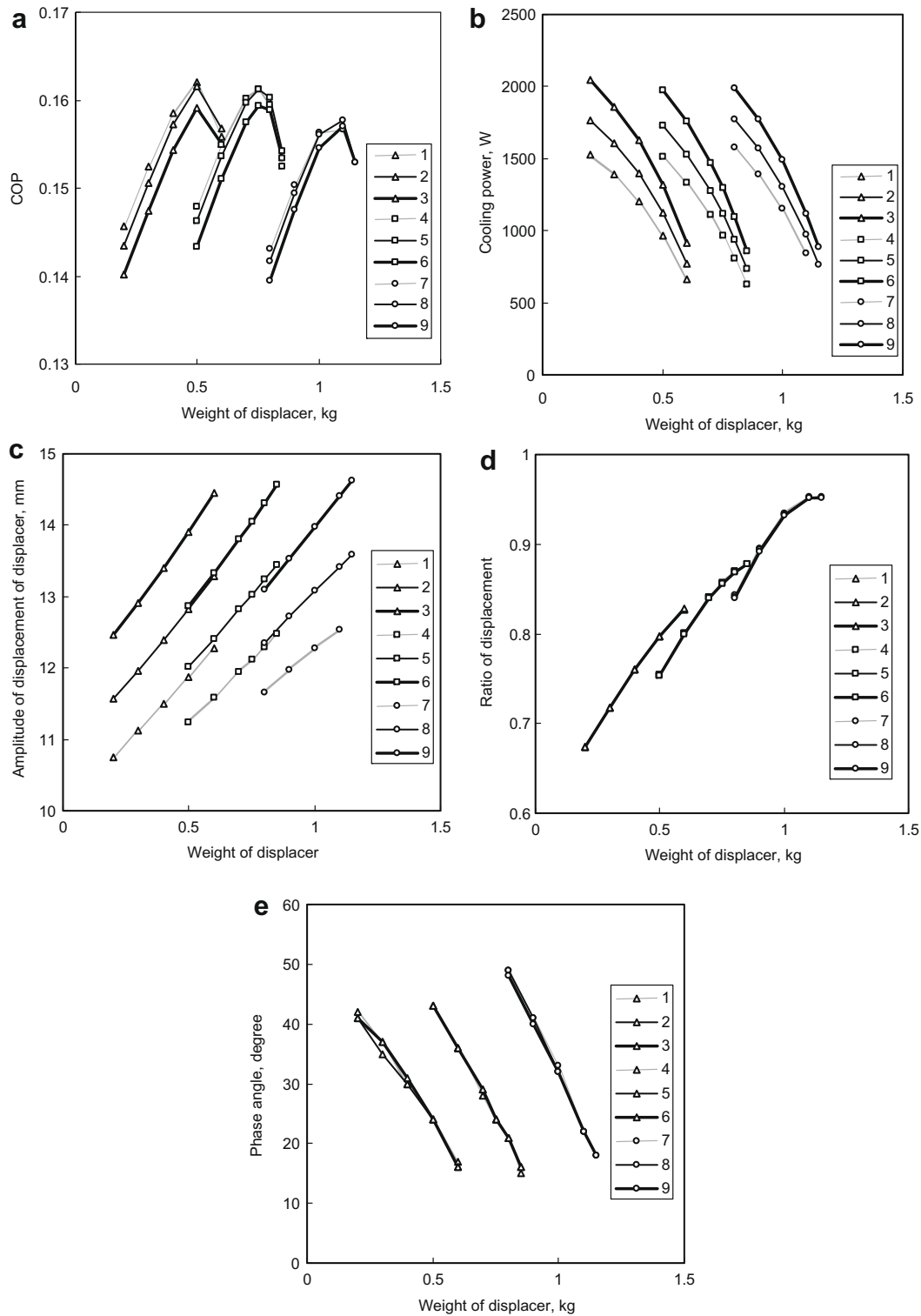


Fig. 9. (a) COP vs. piston weight, displacer weight and rod diameter: 1. rod 30 mm, piston 11 kg; 2. rod 30 mm, piston 11.5 kg; 3. rod 30 mm, piston 12 kg; 4. rod 40 mm, piston 11 kg; 5. rod 40 mm, piston 11.5 kg; 6. rod 40 mm, piston 12 kg; 7. rod 50 mm, piston 11 kg; 8. rod 50 mm, piston 11.5 kg; 9. rod 50 mm, piston 12 kg. (b) Cooling power vs. piston weight, displacer weight and rod diameter: 1. rod 30 mm, piston 11 kg; 2. rod 30 mm, piston 11.5 kg; 3. rod 30 mm, piston 12 kg; 4. rod 40 mm, piston 11 kg; 5. rod 40 mm, piston 11.5 kg; 6. rod 40 mm, piston 12 kg; 7. rod 50 mm, piston 11 kg; 8. rod 50 mm, piston 11.5 kg; 9. rod 50 mm, piston 12 kg. (c) Amplitude of displacement of displacer vs. piston weight, displacer weight and rod diameter: 1. rod 30 mm, piston 11 kg; 2. rod 30 mm, piston 11.5 kg; 3. rod 30 mm, piston 12 kg; 4. rod 40 mm, piston 11 kg; 5. rod 40 mm, piston 11.5 kg; 6. rod 40 mm, piston 12 kg; 7. rod 50 mm, piston 11 kg; 8. rod 50 mm, piston 11.5 kg; 9. rod 50 mm, piston 12 kg. (d) Ratio of displacement vs. piston weight, displacer weight and rod diameter: 1. rod 30 mm, piston 11 kg; 2. rod 30 mm, piston 11.5 kg; 3. rod 30 mm, piston 12 kg; 4. rod 40 mm, piston 11 kg; 5. rod 40 mm, piston 11.5 kg; 6. rod 40 mm, piston 12 kg; 7. rod 50 mm, piston 11 kg; 8. rod 50 mm, piston 11.5 kg; 9. rod 50 mm, piston 12 kg. (e) Phase angle between piston and displacer vs. piston weight, displacer weight and rod diameter: 1. rod 30 mm, piston 11 kg; 2. rod 30 mm, piston 11.5 kg; 3. rod 30 mm, piston 12 kg; 4. rod 40 mm, piston 11 kg; 5. rod 40 mm, piston 11.5 kg; 6. rod 40 mm, piston 12 kg; 7. rod 50 mm, piston 11 kg; 8. rod 50 mm, piston 11.5 kg; 9. rod 50 mm, piston 12 kg.

increases with the phase angle increase, and decreases with the increase in the swept volume ratio of displacer to compressor, which is opposite to that in the refrigerator.

Fig. 7 shows the P - x diagrams in the pulse tube, the pressure wave and mass flow rate at the hot end of the pulse tube, the PV diagrams of compressor and displacer, enthalpy flow, exergy flow, and temperature distribution in the machine at swept volume ratio of 0.6 and phase angle of 20° . The pressure is delayed by a large phase angle with respect to the mass flow rate. There is a sufficient distance between the P - x diagrams. The PV diagram of displacer is larger than that of compressor. In Fig. 7d, the flat line of enthalpy flow indicates the regenerator and the pulse tube. The flat line of temperature indicates the cooler, absorption heat exchanger, and connecting tube. The enthalpy loss in the regenerator is from the hot end to the warm end, so it is negative. The exergy flow in the heater increases, and keeps nearly constant in the pulse tube.

The above results show that there is an optimum phase angle and swept volume ratio for the efficiency of the engine at a fixed compressor swept volume. This is the same when it is used as the refrigerator.

5. Numerical results with linear motor and displacer

In the following section, the mass of the displacer, the diameter of the rod, and the mass of the piston are discussed when the machine is used as the cryogenic refrigerator under the condition that the linear motor and displacer are included in the numerical model. Table 4 shows the calculation condition.

Fig. 8 shows the voltage, current, displacement of piston and displacer, and pressure with the condition shown in Table 4. The amplitudes of the displacement of the piston and the displacer are 16 mm and 13 mm, respectively. The phase angle between the displacement of the displacer and the piston is 24° . The phase angle between the current and the displacement of the piston is 86° , which means that the current and the velocity of the piston are almost in phase, and the motor oscillates near its resonance point. The pressure in the motor house and the buffer is near the average level.

Fig. 9 shows the COP, cooling power, amplitude of displacement of the displacer, amplitude ratio of the displacer to the piston, phase angle between the displacer and the piston change with the mass of the displacer, the diameter of the displacer rod, and the mass of the piston. For each displacer rod diameter, there is an optimum displacer mass for the COP, and the optimum point is hardly influenced by the piston mass. The cooling power increases with the decrease of the displacer mass and with the increase of the piston mass. The amplitude of the displacement of the displacer increases with the increase of the displacer mass and the piston mass. The amplitude ratio of the displacer to the piston has little relation with the mass of the piston, and increases nearly in a linear fashion with the increase of the mass of the displacer. The phase angle between the displacer and the piston is scarcely influenced by the mass of the piston, and almost decreases linearly with the decrease of the mass of the displacer.

At optimum COP point, for rods of 30 mm, 40 mm, and 50 mm, the masses of the displacer are 0.5 kg, 0.75 kg, and 1.1 kg, respectively, the phase angle between the displacer and the piston is

24° , 24° , and 22° , respectively, and the ratios of the amplitude of the displacement of the displacer to the piston are 0.8, 0.86, and 0.95, respectively. This means that a bigger rod diameter gives a bigger amplitude ratio of the displacer to the piston at the optimum point. So the bigger rod diameter has a higher swept volume ratio of the displacer to the piston.

In Fig. 9, the increase of the mass of the displacer means that the natural frequency of the displacer is decreased. So the natural frequency is a strong parameter to adjust the phase angle between the piston and the displacer.

In this section, the numerical results show that the phase angle between displacer and piston can be controlled by the mass of the displacer, and the displacement ratio of the displacer to the piston can be controlled by the rod diameter and the mass of the displacer.

6. Conclusion

We have introduced a nodal analysis numerical model for the simulation of the pulse tube stirling machine with warm gas-driven displacer and a displacer rod. The model was used to simulate a machine when it is used as cryogenic refrigerator, room temperature refrigerator and engine. The results are useful for understanding this type of machine. For given fixed parts and a given swept volume, there is an optimum phase angle between the compressor and displacer for COP when it is used as refrigerator, and for efficiency when it is used as engine. When it is used as cryogenic refrigerator, the mass of the displacer can be used to adjust the phase angle between the compressor and the displacer, and the amplitude ratio of the displacer to the piston when the rod diameter is fixed. At the optimum COP point, the larger rod diameter has the higher amplitude ratio of the displacer to the piston.

References

- [1] Mikulin EI, Tarasov AA, Shkrebyonok MP. Low temperature expansion pulse tubes. *Adv Cryog Eng* 1984;29:629.
- [2] Radebaugh R, Zimmerman J, Smith DR, Louie B. A comparison of three types of pulse tube refrigerators: new method for reaching 60 K. *Adv Cryog Eng* 1986;31:779.
- [3] Zhu SW, Wu P. Double inlet pulse tube refrigerators: an important improvement. *Cryogenics* 1990;30:514–20.
- [4] Tominaga A. Phase controls for pulse tube refrigerator of the third generation. *Cryog Eng* 1992;27(2):147–51.
- [5] Zhu SW, Matsubara Y. Numerical method of inertance tube pulse tube refrigerator. *Cryogenics* 2004;44:649–60.
- [6] Ishizaki Y, Ishizaki E. Prototype of pulse tube refrigerator for practical use. In: *Advances in Cryogenic Engineering*, vol. 39B; 1994. p. 1433–9.
- [7] Inoue T, Kawano S, Aoyama K. Heat flow and temperature distribution in the pulse tube refrigerator. In: *Proceedings of fourth joint Sino-Japanese seminar on cryocooler and concerned topics*, Beijing, PR China, October 19–23; 1993. p. 89–93.
- [8] Radebaugh R. Development of pulse tube refrigerator as an efficient and reliable cryocooler. In: *Proc. Institute of refrigeration*, London; 1999–2000.
- [9] Nogawa M, Zhu SW, Inoue T. Study of capacities on stirling refrigerator with pulse tube. In: *The 5th symposium on stirling cycle*, Japan, October 26; 2001. p. 129–32.
- [10] Matsubara Y. Future trend of pulse tube cryocooler research. In: Liang Zhang, Liangzhen Lin, Guobang Chen, editors. *Proceedings of the twentieth international cryogenic engineering conference*, Beijing, China 2004. Elsevier Ltd.; 2005. p. 189–96.
- [11] Zhu SW. Pulse tube regenerative heat machine, Japan patent application 2005-93277.